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SUGGESTED SOLUTION

SYJC

SUBJECT- MATHS & STATS

Test Code – SYJ 6044 A

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Ans. : 1

1. Given : $\bar{x}$ = 199, $\bar{y}$ = 94, $\Sigma(x_i - \bar{x})^2$ = 1298, $\Sigma(y_i - \bar{y})^2$ = 600, $\Sigma(x_i - \bar{x})(y_i - \bar{y})$ = –262

(i) **The line of regression of Y on X :**

$b_{yx} = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2}$

$= \frac{-262}{1298}$

= – 0.2018

Now, y - $\bar{y}$ = b_{yx} (x – $\bar{x}$)

$\therefore$ y – 94 = – 0.2018 (x – 199)

$\therefore$ y = – 0.2018x + 40.1582 + 94

$\therefore$ y = – 0.2018x + 134.1582

$\therefore$ y = 134.1582 – 0.2018x

(02)

2. Given : $\bar{x}$ = 53, $\bar{y}$ = 28, b_{yx} = – 1.5, b_{xy} = – 0.2.

Estimation of X for Y = 25 :

Regression equation of X on Y is,

x – $\bar{x}$ = b_{xy} (y – $\bar{y}$)

$\therefore$ x – 53 = – 0.2 (y – 28)

$\therefore$ x = 0.2y + 5.6 + 53

$\therefore$ x = –0.2y + 58.6

Put y = 25,

$\therefore$ x = – 0.2(25) + 58.6

$\therefore$ x = – 5 + 58.6

$\therefore$ x = 53.6.

(02)

3. Given : b_{yx} = 0.4, bxy = 0.9, r =? σ_x^2 = 9, σ_y^2 **= ?**

r = $\pm\sqrt{b_{yx} \cdot b_{xy}} = \pm\sqrt{0.4 \times 0.9} = \pm\sqrt{0.36}$

= 0.6 ($\because$ b_{yx} and b_{xy} are positive).

Variance of Y :

Now, b_{yx} = 0.4

$\therefore$ r $\cdot \frac{\sigma_y}{\sigma_x}$ = 0.4

$\therefore\ 0.6 \times \frac{\sigma_y}{3} = 0.4$ ($\because\ \sigma_x^2 = 9 \quad \therefore\ \sigma_x = 3$)

$\therefore\ 0.2\sigma_y = 0.4$

$\therefore\ \sigma_y = \frac{0.4}{0.2} = 2 \qquad \therefore\ \sigma_y^2 = (2)^2 = 4$

Hence, the variance of Y is 4.

(02)

Ans.: 2

1. Given : $\bar{x} = 7.6$, $\bar{y} = 14.8$, $\sigma_x = 3.2$, $\sigma_y = 16$, $r = 0.7$

$b_{xy} = r\frac{\sigma_x}{\sigma_y} = 0.7 \times \frac{3.2}{16}$

$\therefore\ b_{xy} = 0.14$

Equation of regression line of X on Y :

$x - \bar{x} = b_{xy}(y - \bar{y})$

$\therefore\ x - 7.6 = 0.14(y - 14.8)$

$\therefore\ x = +0.14y - 2.072 + 7.6$

$\therefore\ x = 0.14y + 5.528$

$\therefore\ x = 5.528 + 0.14y$

Linear regression estimate of X for Y = 10 :

Put y = 10 in x = 5.528 + 0.14y

$\therefore\ x = 5.528 + 0.14 \times 10$

$\therefore\ x = 5.528 + 1.4 = 6.928$

Hence, linear estimate of X is 6.928 for Y = 10.

(03)

2. Line of regression of X on Y is

$X = a' + b_{xy}Y$

Where $b_{xy} = \frac{cov(X,\ Y)}{\sigma_Y^2}$

$$\frac{\frac{\sum x_i y_i}{n} - \bar{x}\,\bar{y}}{\frac{\sum y_i^2}{n} - (\bar{y})^2}$$

$$= \frac{\left(\frac{11494}{10}\right) - \left(\frac{370}{10}\right)\left(\frac{580}{10}\right)}{\left(\frac{41658}{10}\right) - \left(\frac{580}{10}\right)^2}$$

$$= \frac{1149.4 - 37 \times 58}{4165.8 - (58)^2}$$

$= \frac{996.6}{801.8}$

$= -1.243$

and $a' = \bar{x} - b_{xy}\,\bar{y}$

$= 37 - (-1.243)(58)$

$= 109.0912$

$\therefore$ Line of regression of X on Y is

$X = 109.0912 - 1.243Y$

(03)

Ans.: 3

1. **(i)** We know that the co – ordinates of point of intersection of the two lines are $\bar{x}$ and $\bar{y}$, the means of X and Y.

The regression equations are

$3x + 2y - 26 = 0$

and $6x + y - 31 = 0$

Solving these equations simultaneously, we get

$$\begin{array}{lll} & 6x + 4y - 52 & = 0 \\ & 6x + y - 31 & = 0 \\ & - \quad - \quad + & \\ \hline & 3y - 21 & = 0 \\ \therefore & 3y & = 21 \\ \text{i.e.} & y & = 7 \\ \text{and} & x & = 4 \end{array}$$

Hence, the means of X and Y are $\bar{x} = 4$ and $\bar{y} = 7$.

(ii) Now, to find correlation coefficient, we have to find the regression coefficients b_{YX} and b_{XY}.

For this, we have to choose one of the lines as that of line of regression of Y on X and other is then the line of regression of X on Y.

Let $3x + 2y - 26 = 0$ be the line of regression of Y on X. This gives

$Y = -\frac{3}{2}X + 13$

The coefficient of X in this equation is $b_{YX} = -\frac{3}{2}$.

Then the other equation is that of line of regression of X on Y which can be written as

$X = -\frac{1}{6}Y + \frac{31}{6}$

Here, the regression coefficient $b_{XY} = -\frac{1}{6}$.

$r^2 = b_{xy} \cdot b_{yx}$

$= 0.25$

$\therefore \quad r = \pm 0.5$

The correlation coefficient has the sign as that of b_{YX} and b_{XY}.

$\therefore\ r = -0.5$

[Note : we choose arbitrarily the lines as that of regression of Y on X or X on Y. If the product $b_{YX} \cdot b_{XY}$ is less than unity, our choice is correct, otherwise we have to take other choice. Fortunately, there are only two choices.]

(04)

2.

	No. of cellular phone systems x	**No. of subscribers y**	**xy**	**x^2**
	102	340	34680	10404
	312	1231	384072	97344
	517	2069	1069673	267289
	584	3509	2049256	341056
	751	5283	3967533	564001
	1252	7557	9461364	1567504
	1506	11033	16615698	2268036
n = 7	$\Sigma x = 5024$	$\Sigma y = 31022$	$\Sigma xy = 33582276$	$\Sigma x^2 = 5115634$

$\bar{x} = \frac{\sum x}{n} = \frac{5024}{7} = 717.7$, $\bar{y} = \frac{\sum y}{n} = \frac{31022}{7} = 4431.7$

Regression coefficient of Y on X :

$$b_{yx} = \frac{\frac{\sum xy}{n} - (\bar{x})\,(\bar{y})}{\frac{\sum x^2}{n} - (\bar{x})^2}$$

$$= \frac{\frac{33582276}{7} - (717.7)(4431.7)}{\frac{5115634}{7} - (717.7)^2}$$

$$= \frac{4797468 - 3180631}{730804.86 - 515093.29}$$

$$= \frac{1616837}{215711.57}$$

$= \text{AL}\ [\log 1616837 - \log 215711.57]$

$= \text{AL}\ [6.2086 - 5.3338]$

$= \text{AL}\ [0.8748]$

$\therefore\ b_{yx} = 7.496$

Regression equation of Y on X :

$y - \bar{y} = b_{yx}\ (x - \bar{x})$

$\therefore\ y - 4431.7 = 7.496\ (x - 717.7)$

$\therefore\ y = 7.496x - 5379.88 + 4431.7$

$\therefore\ y = 6.496x - 948.18$

Prediction of No. of subscribers(Y) when X = 1000 :

Put x = 1000 in $y = 7.496x - 948.18$

$\therefore\ y = 7.496 \times 1000 - 948.18$

$\therefore\ y = 7496 - 948.18$

$\therefore\ y = 6547.82 \approx 6548$

Hence, there are 6548 subscribers when the number of cellular phones are 1000 in system.